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APPROXIMATE ESTIMATES OF THE HIGH-TEMPERATURE CREEP OF STRUCTURAL ELEMENTS

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The creep and long-term strength of structural elements under temperatures much higher than operational temperatures are studied. The actual nonuniform stress–strain state is reduced to a fictitious uniform state by averaging the specific dissipated power over the volume of the body. This allows one to estimate the intensity of creep and long-term strength of structural elements in terms of volume-averaged energy parameters by using methods of ideal plasticity and considering statically possible stress fields and kinematically possible velocity and strain rate fields.

1. Basic Relations. We consider a uniformly heated body loaded by volume (F_i) and surface (T_i) forces. For simplicity, external thermal and force parameters are assumed to be stationary. At high temperatures, the stresses σ_{ij}, creep-strain rates η_{ij}, and specific dissipated power $W = \sigma_{ij}\eta_{ij}$ at each point of the body reach stationary values in a short time. Since σ_{ij} and η_{ij} are related by an equation of the type $\eta_{\rm eq} = f(\sigma_{\rm eq}, T)$ ($\eta_{\rm eq}$ and $\sigma_{\rm eq}$ are the equivalent strain rate and equivalent stress, respectively, which are functions of the invariants of their tensors), the dissipated power can be expressed in terms of stresses $W = \Phi(\sigma_{ij})$ or creep-strain rates $W = U(\eta_{ij})$ in an invariant form. It is common practice to use the intensities of the corresponding tensors as equivalent quantities:

$$\sigma_{\rm eq} = (3\sigma^0_{ij}\sigma^0_{ij}/2)^{1/2}, \qquad \eta_{\rm eq} = (2\eta^0_{ij}\eta^0_{ij}/3)^{1/2}.$$

Here $\sigma^0_{ij} = \sigma_{ij} - \delta_{ij}\sigma_{nn}/3$ and $\eta^0_{ij} = \eta_{ij} - \delta_{ij}\eta_{nn}/3$ are the deviator tensors ($\eta^0_{ij} = \eta_{ij}$ for plastic incompressibility).

At any point of the body, one can determine the dissipated power $W = \Phi(\sigma_{ij})$ or $W = U(\eta_{ij})$ and study the effect of external force parameters on W in the space of stresses or strain rates. In particular, if the dissipated power remains unchanged, $\Phi(\sigma_{ij}) = \text{const}$ and $U(\eta_{ij}) = \text{const}$ in the corresponding spaces are determined by surfaces of constant dissipated energy. Assuming that the surfaces are convex, we obtain the Mises criterion

$$(\sigma_{ij} - \sigma^*_{ij})\eta_{ij} \geqslant 0 \qquad \text{or} \qquad (\eta_{ij} - \eta^*_{ij})\sigma_{ij} \geqslant 0, \tag{1}$$

and vice versa, inequalities (1) imply that the surfaces $\Phi(\sigma_{ij}) = \text{const}$ and $U(\eta_{ij}) = \text{const}$ are convex. In the theory of steady creep, these surfaces play the same role as the yield surface in the theory of ideal plasticity [1, 2].

We express the dissipated power averaged over the volume of the body in terms of stresses or strain rates:

$$W^0 = \frac{1}{V}\int\limits_V \sigma_{ij}\eta_{ij}\,dV = \frac{1}{V}\int\limits_V \Phi(\sigma_{ij})\,dV = \Phi(\hat\sigma_{ij}),$$

or

$$W^0 = \frac{1}{V}\int\limits_V \sigma_{ij}\eta_{ij}\,dV = \frac{1}{V}\int\limits_V U(\eta_{ij})\,dV = U(\hat\eta_{ij}). \tag{2}$$

If the points $A(\hat\sigma_{ij})$ and $B(\hat\sigma^*_{ij})$ corresponding to the average stress state of the body lie on the surface $\Phi(\hat\sigma_{ij}) = \text{const}$ in the stress space, the stress states $\hat\sigma_{ij}$ and $\hat\sigma^*_{ij}$ can refer to different physical points of the body (this is also true for the strain rates $\hat\eta_{ij}$ and $\hat\eta^*_{ij}$ in the strain-rate space). In [3–7], a so-called characteristic point is used in approximate calculations of structural elements in creep. (It is assumed that there exists a point whose stress–strain state

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characterizes the behavior of the entire structure.) Thus, it is expedient to consider a "characteristic stress or strain rate," which gives integral estimates of the stress–strain state of the whole body for a chosen measure, say, the average dissipated power rather than the "characteristic point," which is not a fixed point in the body as was shown in [3–7].

We assume that the surfaces $\Phi(\hat{\sigma}_{ij}) = \text{const}$ and $U(\hat{\sigma}_{ij}) = \text{const}$ are convex and the Mises criterion (1) is valid for the averaged values of $\hat{\sigma}_{ij}$ and $\hat{\eta}_{ij}$:

$$(\hat{\sigma}_{ij} - \hat{\sigma}^*_{ij})\hat{\eta}_{ij} \geqslant 0, \qquad (\hat{\eta}_{ij} - \hat{\eta}^*_{ij})\hat{\sigma}_{ij} \geqslant 0. \tag{3}$$

Using statically possible stresses and kinematically possible strain rates in relations (2) and (3), one can estimate approximately the intensity of creep processes and long-term strength of a structural element in the average over its volume.

Statically Possible Stress Fields. Let the volume external loads F_i and surface external loads T_i produce the stresses σ_{ij}, displacement velocities v_i, and corresponding strain rates η_{ij} in the body. Equating the powers of external and internal forces, we obtain

$$\frac{1}{V}\bigg(\int\limits_V F_i v_i\,dV + \int\limits_S T_i v_i\,dS\bigg) = \frac{1}{V}\int\limits_V \sigma_{ij}\eta_{ij}\,dV = \Phi(\hat{\sigma}_{ij}). \tag{4}$$

We consider statically possible stresses σ^*_{ij} that satisfy the equations of equilibrium, boundary conditions, and additional condition of constant average dissipated power W^0:

$$W^0 = \frac{1}{V}\int\limits_V \Phi(\sigma^*_{ij})\,dV = \Phi(\hat{\sigma}^*_{ij}),$$

i.e., the points $A(\hat{\sigma}_{ij})$ and $B(\hat{\sigma}^*_{ij})$ in the stress space lie on the surface $\Phi(\hat{\sigma}_{ij}) = \text{const}$ that corresponds to the true value of the average dissipated power W^0. From the viewpoint of classical mechanics, this means that the surface $\Phi(\hat{\sigma}_{ij}) = \text{const}$ is the ideal constraint. The points in the stress space that correspond to the stress state cannot leave this surface. For the transition from the point $A(\hat{\sigma}_{ij})$ to the point $B(\hat{\sigma}^*_{ij})$, the virtual-work principle yields

$$\hat{\eta}_{ij}\delta\hat{\sigma}_{ij} = \frac{1}{V}\bigg(\int\limits_V \delta F_i v_i\,dV + \int\limits_S \delta T_i v_i\,dS\bigg). \tag{5}$$

Substituting the expressions $\delta\hat{\sigma}_{ij} = \hat{\sigma}^*_{ij} - \hat{\sigma}_{ij}$, $\delta F_i = F^*_i - F_i$, and $\delta T_i = T^*_i - T_i$, into Eq. (5) and taking into account (3), we obtain

$$\int\limits_V F^*_i v_i\,dV + \int\limits_S T^*_i v_i\,dS \leqslant \int\limits_V F_i v_i\,dV + \int\limits_S T_i v_i\,dS. \tag{6}$$

Hence, statically possible stresses that satisfy the condition of constant average dissipated power correspond to smaller external loads compared to the actual stresses. In particular, if the external loads are reduced to the generalized force Q and $\dot{q}$ is the corresponding generalized velocity $\Big(\int\limits_V F_i v_i\,dV + \int\limits_S T_i v_i\,dS = Q\dot{q}\Big)$, we obtain the lower bound of the external forces $Q^*_s \leqslant Q$ as in the case of ideal plasticity.

Kinematically Possible Strain-Rate Fields. In addition to the actual fields of stresses σ_{ij}, velocities v_i, and strain rates η_{ij}, we consider a kinematically possible velocity field v^*_i, which is assumed to be continuous and differentiable to obtain a kinematically possible strain-rate field η^*_{ij}. Moreover, we require that the kinematically possible field η^*_{ij} satisfies the condition of conservation of the average dissipated power $W^0 = U(\hat{\eta}^*_{ij}) = \text{const}$ corresponding to the true values of σ_{ij} and η_{ij}. In this case, relations (2) imply that the stress field σ^*_{ij} corresponding to η^*_{ij} also satisfies the condition $W^0 = \Phi(\hat{\sigma}^*_{ij}) = \text{const}$, i.e., the point $C(\hat{\sigma}^*_{ij})$ in the stress space lies on the same surface $\Phi(\hat{\sigma}_{ij}) = \text{const}$ as the point $A(\hat{\sigma}_{ij})$ with true values of stresses. However, the quantities σ^*_{ij} and η^*_{ij} correspond to different external loads F^*_i and T^*_i and velocities v^*_i, i.e., we obtain the equation

$$\frac{1}{V}\bigg(\int\limits_V F^*_i v^*_i\,dV + \int\limits_S T^*_i v^*_i\,dS\bigg) = \frac{1}{V}\int\limits_V \sigma^*_{ij}\eta^*_{ij}\,dV = \Phi(\hat{\sigma}^*_{ij}) \tag{7}$$

instead of (4). Since $\Phi(\hat{\sigma}^*_{ij}) = \Phi(\hat{\sigma}_{ij})$, the left sides of Eqs. (4) and (7) are equal.

Let $\delta\hat{\sigma}_{ij}$ be the displacement from the point of the possible state $C(\hat{\sigma}^*_{ij})$ to the point of the actual state $A(\hat{\sigma}_{ij})$ over the surface $W^0 = \Phi(\hat{\sigma}^*_{ij}) = \text{const}$ (ideal constraint).

Similarly to (5), we obtain

$$\hat{\eta}^*_{ij}\delta\hat{\sigma}_{ij} = \frac{1}{V}\Bigg(\int\limits_V \delta F_i v^*_i\,dV + \int\limits_S \delta T_i v^*_i\,dS\Bigg). \tag{8}$$

Substituting $\delta\hat{\sigma}_{ij} = \hat{\sigma}_{ij} - \hat{\sigma}^*_{ij}$, $\delta F_i = F_i - F^*_i$, and $\delta T_i = T_i - T^*_i$ into Eq. (8), with allowance for (3), we have

$$\int\limits_V F_i v^*_i\,dV + \int\limits_S T_i v^*_i\,dS \leqslant \int\limits_V F^*_i v^*_i\,dV + \int\limits_S T^*_i v^*_i\,dS = \int\limits_V F_i v_i\,dV + \int\limits_S T_i v_i\,dS. \tag{9}$$

Thus, kinematically possible fields of displacement velocities and strain rates that satisfy the condition of constant average dissipated power correspond to higher external loads compared to the actual fields. Inequality (9) can also be formulated as follows: for kinematically possible velocities v^*_i, actual external loads F_i and T_i give a smaller value of the average dissipated power compared to the actual velocities v_i, i.e., the loads F^*_i and T^*_i must exceed the actual loads to ensure the same value of W^0. In this formulation, inequalities (6) and (9) are similar to the principles of minimum power and minimum complementary dissipation in the theory of steady creep [1, 8]. Moreover, if the external loads are reduced to the generalized force Q and the corresponding generalized velocity $\dot{q}$, for example, $\int\limits_V F_i v^*_i\,dV + \int\limits_S T_i v^*_i\,dS = Q\dot{q}^*$ and $\int\limits_V F^*_i v^*_i\,dV + \int\limits_S T^*_i v^*_i\,dS = Q^*_k\dot{q}^*$, we obtain the upper bound of external forces $Q \leqslant Q^*_k$. As in the case of ideal plastic structures, we estimate the lower and upper bounds of external loads:

$$Q^*_s \leqslant Q \leqslant Q^*_k. \tag{10}$$

These estimates of external loads can be used, for example, in technological problems of material processing by pressure under creep and superplasticity conditions, where only information on the power of production equipment is needed, and exact values of stresses and strain rates in a workpiece are out of interest.

In contrast to the problems of ideal plasticity, where it is usually required to determine limit external loads acting on a structural element and obtain the upper and lower bounds of these quantities, another class of problems is of interest in applied problems of creep, where the intensity of creep with a certain measure and the rupture time of the structural element should be estimated for given external loads. In the energy variant of creep [9, 10], the specific dissipated power $W = \sigma_{ij}\eta_{ij}$ is used as a measure of intensity of the process, and the dissipated specific energy $A = \int\limits_0^t \sigma_{ij}\eta_{ij}\,dt$ is used as a measure of damage of the material. It is shown that the rupture time t^* of the material in creep is inversely proportional to the quantity W at the steady stage of the process: $W_l t^*_l = W_n t^*_n$. This is experimentally supported for a uniform stress–strain state of a number of structural materials under stationary and nonstationary external conditions in a wide range of thermal and force parameters. Assuming that these statements are satisfied "in the average" for structural elements with a nonuniform stress–strain state, from the above-given inequalities of the type (10), we obtain two inequalities:

1) For constant thermal and force parameters, the use of statically possible stress fields σ^*_{ij} gives the upper bound of the volume-averaged specific dissipated energy W^0, whereas the use of kinematically possible velocities v^*_i and strain rates η^*_{ij} gives the lower bound

$$W^0_s \geqslant W^0 \geqslant W^0_k; \tag{11}$$

2) For statically and kinematically possible fields, the rupture time t^* of a structural material is inversely proportional to the quantities W^0 "in the average", and with allowance for (11), we obtain

$$t^*_s \leqslant t^* \leqslant t^*_k. \tag{12}$$

Below, we consider the results of physical and numerical experiments to estimate the behavior of some typical structural elements in high-temperature creep and verify the above inequalities.

2. Characteristics of the High-Temperature Creep of the Material Used. An St. 45 iron-based structural alloy was used in experiments. The material was used in its delivery state, and no thermal treatment of experimental specimens was performed before and after fabrication. The creep of cylindrical specimens in uniaxial tension and compression was studied. The specimens had the following dimensions: the working length

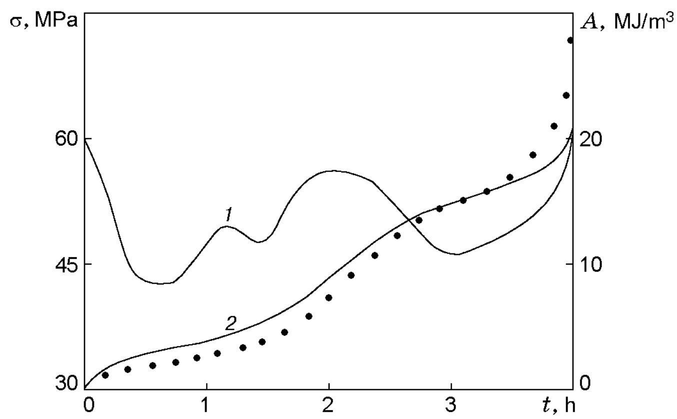

Fig. 1

and diameter were $l_0 = 37$ mm and $d_0 = 10$ mm, respectively, in tension experiments and $l_0 = 20$ mm and $d_0 = 7$ mm in compression experiments. The specimens were loaded by tensile or compressive forces until failure or buckling occurred, respectively. To ensure constant stresses in the specimen ($\sigma = \text{const}$) with a 1% change in strain, we corrected the external load using the condition of plastic incompressibility $S_0 l_0 = Sl$, where $S(t)$ and $l(t)$ are the current cross-sectional area and length of the specimen, respectively.

To determine characteristics of the material, we performed experiments for fixed and varied values of σ. The tests were performed at a temperature $T = 725°$C, which was sustained within $\pm 0.5\%$ of the specified value.

We constructed dependences of the dissipated work done by the creep strains $A = \sigma\varepsilon$ on the time t, where $\varepsilon = \ln(l(t)/l_0)$. The distinct linear character of the curves allowed us to determine the specific dissipated power $W_0 = dA/dt = \sigma\eta$. It was found that the strain rates in tension and compression experiments were very close for the same σ.

Processing the experimental data, we obtained the following approximating relations of the form [11]:

$$W = B\sigma_{\text{eq}}^n, \quad \sigma_{\text{eq}} = (3\sigma_{ij}^0\sigma_{ij}^0/2)^{1/2}, \quad \sigma_{ij}^0 = \sigma_{ij} - \delta_{ij}\sigma_{nn}/3, \quad B = 3.5 \cdot 10^{-14}, \quad n = 6.22; \tag{13}$$

$$W = D\eta_{\text{eq}}^m, \quad \eta_{\text{eq}} = (2\eta_{ij}^0\eta_{ij}^0/3)^{1/2}, \quad \eta_{ij}^0 = \eta_{ij}, \quad D = 376.88, \quad m = 1.19. \tag{14}$$

In (13) and (14), W is in MJ/(m$^3 \cdot$ sec), B is in MPa^{1-n}/sec, and D is in MPa $\cdot$ sec^{m-1}. One can take into account the softening stage by writing the above relations in the form

$$\frac{dA}{dt} = \frac{(A^*)^s B\sigma_{\text{eq}}^n}{(A^* - A)^s} \qquad \text{or} \qquad \frac{dA}{dt} = \frac{(A^*)^s D\eta_{\text{eq}}^m}{(A^* - A)^s}, \tag{15}$$

where the experimental values $A^* = 30$ MJ/m^3 (the energy dissipated by the moment of failure) and $s = 0.9$ are used. As $A \to 0$, relations (15) become (13) and (14). We have $\sigma_{\text{eq}} = \sigma$ and $\eta_{\text{eq}} = \eta$ for a bent beam and $\sigma_{\text{eq}} = \sqrt{3}\tau$ and $\eta_{\text{eq}} = (\sqrt{3}/3)\dot{\gamma}$ for a twisted bar (τ is the shear stress and γ is the shear strain). It should be noted that a neck is formed in specimens for strains $\varepsilon \geqslant 12\%$, and the softening exponent s in (15) depends on both the material strength and specimen geometry. Experimental diagrams of deformation of a material, especially in the case of twisted bars, were usually described by relation (13) without allowance for softening. The long-term strength of structural elements was determined by relation (15).

The validity of the above relations was verified experimentally under stationary and nonstationary loading. Figure 1 shows the results of one experiment. Curve 1 refers to the specified relation $\sigma(t)$, the points to the experimental relation $A(t)$, and curve 2 to the relation $A(t)$ calculated by (13).

3. Experimental Results on Approximate Estimation of Creep of Typical Structural Elements. The validity of inequalities (10)–(12) was verified by estimating the intensity of creep of bent beams of rectangular cross section and twisted cylindrical bars.

The experimental curve of the specific energy $A_0 = A(t)$ dissipated in the irreversible deformation of a rectangular beam bent by a constant moment $M = \text{const}$ was determined by the formula $A_0 = M\varphi/V$, where the angle of rotation of the end cross sections of the beam φ was calculated by the approximate formula $\varphi = 8\Delta/L$, in which $L = 100$ mm is the working length of the beam and Δ is the experimentally measured deflection of the beam. Finally, we obtain $A_0 = 8M\Delta/(bhL^2)$, where b and h are the width and height of the beam, respectively.

TABLE 1

p	M_s^*, N · m	d	M_k^*, N · m
0	25.7	0	25.7
0.2	26.6	0.2	26.4
0.7	24.9	1.0	26.6
1.0	23.5	3.3	25.5

TABLE 2

p	M_s^{**}, N · m	d	M_k^{**}, N · m
0	52.4	0	52.4
0.5	52.2	1.0	53.1
1.0	49.3	5.0	49.6

For cylindrical bars twisted by a constant torque $M = \text{const}$, we obtain $A_0 = M\varphi/(\pi R^2 L)$, where the twist angle φ over the working length L was measured directly in experiments.

As in the case of uniaxial processes with a uniform stress–strain state, the curves $A_0 = A(t)$ are characterized by a weakly pronounced section corresponding to the initial unsteady stage; most of the time, the process occurred in accordance with laws of viscous flow without hardening and softening, which allowed us to determine the dissipated power $W_0 = dA/dt$ averaged over the volume of the body. We performed two series of experiments: under stationary thermal and force conditions ($T = 725°$C and $M = \text{const}$) and for arbitrarily varied values of $M = M(t)$.

Estimate of the Creep Intensity for a Constant Moment. We consider the statically possible stress fields in the beam $\sigma(z) = \sigma_0(2z/h)^p$ for $p \geqslant 0$, where σ_0 is the stress in the extreme fibers of the beam for $z = h/2$ (it is assumed that the behavior of the material in tension and compression is the same). By virtue of symmetry, the distribution of $\sigma(z)$ ensures a zero axial force for any p. Further, we require that the average dissipated power be constant for any p, i.e., $W_0 = \text{const}$. From (13), we obtain

$$W_0 = \frac{1}{V}\int\limits_V B\sigma_{\text{eq}}^n \, dV = \frac{2}{h}\int\limits_0^{h/2} B\Big[\sigma_0\Big(\frac{2z}{h}\Big)^p\Big]^n \, dz = \frac{B\sigma_0^n}{pn+1}.$$

Given the value of W_0, we determine σ_0 and distribution of $\sigma(z)$ over the cross section for any p. It remains to satisfy the last condition of equilibrium $M = \int\limits_S \sigma(z) z \, dS$, which relates the external moment to p for different statically possible stress fields. For $\sigma(z)$ given at every point over the beam height, one determines $W(z)$ and $\eta(z) = W(z)/\sigma(z)$ using (13).

Similarly, we consider the kinematically possible strain-rate fields $\eta(z) = \eta_0(2z/h)^d$ for $d \geqslant 0$. For an arbitrary d, the strain-rate compatibility condition may fail, which requires the introduction of shear velocities and the associated shear stresses. Within the approximate estimates, we assume that their contribution to the power W_0 is small compared to the contribution of the longitudinal strain rate $\eta(z)$, i.e., $\eta_{\text{eq}} = \eta(z)$. Inserting $\eta(z)$ into (14), we obtain

$$W_0 = \frac{1}{V}\int\limits_V D\eta_{\text{eq}}^m \, dV = \frac{D\eta_0^m}{dm+1}.$$

As in the previous case, we determine η_0 and $\eta(z)$ for a given value of W_0 and an arbitrary d. Using (14), we find the distribution $W(z)$ over the beam height and $\sigma(z) = W(z)/\eta(z)$.

As was stated previously, for different values of p and d, the z_*-coordinates of the point, where the condition $B[\sigma(z_*)]^n = D[\eta(z_*)]^m = W_0$ holds with allowance for (13) and (14), are different. Therefore, it makes no sense to talk of a characteristic physical point.

The estimates of the bending moments for the above-considered cases as functions of p and d are listed in Table 1. These results were compared with experimental data for a beam ($b = 9.96$ mm, $h = 19.98$ mm, $L = 100$ mm, and $M = 25.83$ N · m). It was found experimentally that $W_0 = 2.08 \cdot 10^{-5}$ MJ/(m^3 · sec). With allowance for scatter of experimental data and the fact that relations (13) and (14) are approximate within the indicated intervals of p and d, inequality (10) holds satisfactorily, and the deviation of the calculated values of M_s^* and M_k^* from the experimental value $M = 25.83$ N · m does not exceed several percent.

Similar results were obtained for a cylindrical bar twisted by the torque M. Statically possible stress fields were taken in the form $\sigma_{\text{eq}} = \sigma_0(r/R)^p$, where $\sigma_0 = \sqrt{3}\tau_0$. Substituting σ_{eq} into (13) and integrating over the working volume of the bar, after averaging we obtain $W_0 = 2B\sigma_0^n/(np+2)$. Specifying W_0 for any p, we find σ_0, $\sigma_{\text{eq}}(r)$, $W(r)$, $\eta_{\text{eq}}(r) = W(r)/\sigma_{\text{eq}}(r)$, and $M_s^{**} = \int\limits_S \tau r \, dS = 2\pi\sigma_0 R^3/(\sqrt{3}(p+3))$.

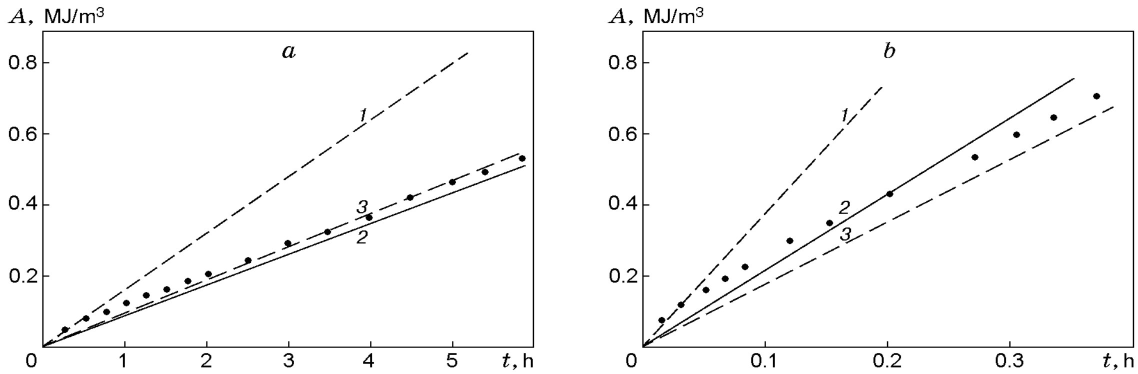

Fig. 2

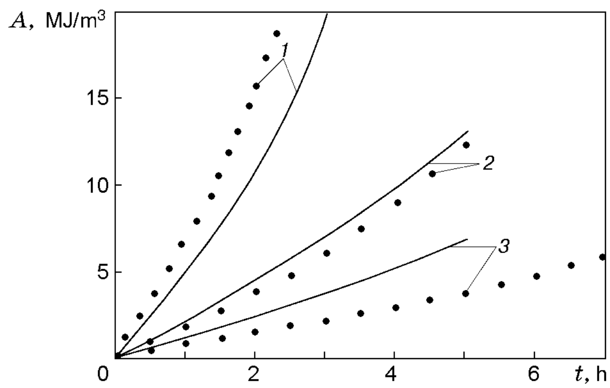

Fig. 3

For kinematically possible strain-rate fields, we assume that $\eta_{\text{eq}} = \eta_0 (r/R)^d$ and $\eta_0 = (\sqrt{3}/3)\dot{\gamma}_0$. Using (14), after integration and averaging, we obtain $W_0 = 2D\eta_0^m/(md+2)$. For specified W_0 and an arbitrary exponent d, we find η_0, $\eta_{\text{eq}}(r)$, $W(r)$, $\sigma_{\text{eq}}(r) = W(r)/\eta_{\text{eq}}(r)$, and M_k^{**}.

Table 2 lists M_s^{**} and M_k^{**} as functions of p and d. The results were compared with experimental data ($R = 10$ mm, $L = 37$ mm, and $M = 53.15$ N $\cdot$ m). It was found experimentally that $W_0 = 5.13 \cdot 10^{-4}$ MJ/(m$^3 \cdot$ sec). As in the case of bending, the calculation results differ from the experimental data by several percent.

The experimental data for bending and torsion show that relations (11) and (12) can be used to estimate the intensity of creep and rupture time of bent beams and twisted bars.

Let the temperature and the bending moment M be specified and constant. The experimental curve of the dissipated energy $A_0 = 8M\Delta/(bhL^2)$ has already been constructed. We consider an arbitrary statically possible stress field $\sigma = \sigma(z)$ determined by the specified moment M. Inserting $\sigma(z)$ into (13), integrating, and averaging over the working volume of the beam, we find W_0. Confining our attention to the steady creep stage (without consideration of the third stage), we obtain the relation $A(t) = W_0 t$ and compare it with the experiment. Considering kinematically possible strain rates and assuming that the distribution of strain rates through the height of the beam is linear, we obtain $\eta(z) = \dot{æ} z$, where $æ = 8z\Delta/L^2$ is the beam curvature and the quantity Δ is determined experimentally. Substitution of $\eta(z)$ into (14), integration, and averaging over the volume yield W_0 and $A_0(t) = W_0 t$.

Figure 2 shows the results of two experiments on bending of beams for $b_1 = 19.85$ mm, $h_1 = 9.85$ mm, and $M_1 = 21.32$ N $\cdot$ m (Fig. 2a) and $b_2 = 9.87$ mm, $h_2 = 19.84$ mm, and $M_2 = 25.53$ N $\cdot$ m (Fig. 2b). The points correspond to the experimental relations $A_0 = A(t)$. Curves 1 refer to the relations $A_0^{(1)} = W_0^{(1)} t$, where $W_0^{(1)}$ is calculated for the statically possible field that corresponds to the elastic distribution of $\sigma(z)$ and does not change during creep. Curves 2 refer to the values of $W_0^{(2)}$ calculated for the statically possible field $\sigma(z) = \text{const}$ in the ideally plastic case; curves 3 refer to the values of $W_0^{(3)}$ calculated for the kinematically possible strain rates $\eta(z)$ distributed linearly over the height of the beam. These results agree well with the approximate estimates (11).

The same result was obtained in experiments on twisted cylindrical bars. In Fig. 3, the points refer to the experimental dependences $A_0 = M\varphi/V$ obtained for specimens with the parameters $R_1 = 10$ mm, $L_1 = 37.59$ mm,

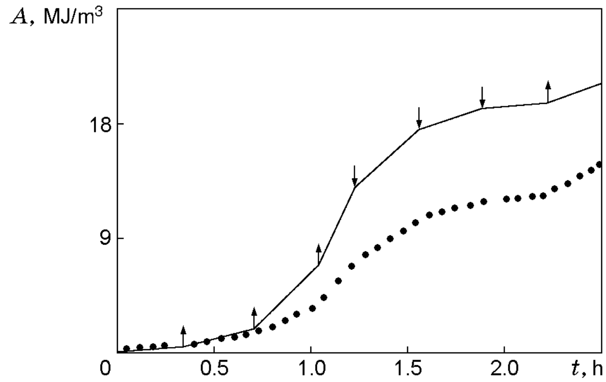

Fig. 4

and $M_1 = 60.37$ N · m, $R_2 = 10$ mm, $L_2 = 47.5$ mm, and $M_2 = 53.2$ N · m, and $R_3 = 10$ mm, $L_3 = 47.5$ mm, and $M_3 = 48.3$ N · m. As in the case of beams, the approximate values of $A_0 = A(t)$ were calculated for statically possible stresses and strain rates. As statically possible stresses, we used the stresses $\sigma_{\rm eq} = \sigma_0(r/R)$ and $\sigma_{\rm eq} = \text{const}$ corresponding to the elastic and limit states, respectively. Given M, we find $\sigma_{\rm eq}(r)$ and W_0, which are assumed to be unchanged from the beginning of creep to failure. The solid curves in Fig. 3 correspond to the theoretical dependences $A = W_0 t$ for the stress fields $\sigma_{\rm eq} = \text{const}$, where W_0 was determined from (15).

As a whole, the experimental data on creep of bent beams and twisted bars under stationary external thermal and force conditions support estimates (10)–(12). To describe the creep, one can use the statically possible stress field corresponding to an ideal plastic body ("ideal creep body" according to the terminology of [8]).

Estimate of the Creep Intensity for Varying Moment. The above dependences were obtained under the assumption of stationary external conditions. However, they can also be used in the case of slowly varying external loads and temperature. At high temperatures, the relaxation time τ^*, i.e., the time it takes for internal thermodynamic parameters to pass to an equilibrium state, is small. If the process is divided into time intervals Δt_i for which $\tau^* \ll \Delta t_i$, the above-considered dependences are valid within each interval. This assumption is supported by experiments on bent beams and twisted continuous cylindrical bars subjected to time-varying moments at a constant temperature. Figure 4 shows the results of one of these experiments. A continuous cylindrical bar with working length $L = 39$ mm and radius $R = 10$ mm was twisted by a torque that varied in a stepwise manner with $\Delta t_i \approx 20$ min (except for $\Delta t_4 \approx 10$ min). The torques were as follows: $M_1 = M_7 = 48.4$ N · m, $M_2 = M_6 = M_8 = 60.5$ N · m, $M_3 = M_5 = 72.5$ N · m, and $M_4 = 84.6$ N · m. The points refer to the experimental dependence $A = A(t)$, and the arrows indicate the overload time. The total dissipated energy in the experiment was determined as $A = \sum\limits_{i=1}^{8} M_i \varphi_i$. The average dissipated power W_0 at each interval Δt_i for fixed moments was calculated from relation (13), in which the stress field corresponding to the limit state (for an ideal creep body) was used as a statically possible stress field. For the values of torque M_i considered, the equivalent stresses $\sigma_{\rm eq} = \sqrt{3}\tau$ were $\sigma_1 = \sigma_7 = 40$ MPa, $\sigma_2 = \sigma_6 = \sigma_8 = 50$ MPa, $\sigma_3 = \sigma_5 = 60$ MPa, and $\sigma_4 = 70$ MPa. The solid curve in Fig. 4 shows the theoretical dependences $A(t)$ determined as a sum $A = \sum\limits_{i=1}^{8} W_0^i \Delta t_i$. As in the stationary case, the use of statically possible stresses in calculations yields overestimated values of the dissipated power compared to experimental data but, as a whole, satisfactorily describes the deformation.

It is noteworthy that the slopes of experimental curves are almost the same in the intervals Δt_i where the moments M_i are equal. This supports the validity of the above-adopted assumptions of a small relaxation time of a material τ^*, fast transition from a perturbed state caused by overload to an equilibrium state, and applicability of the above estimates to nonstationary processes.

4. Estimate of the Rupture Time of Structural Elements in High-Temperature Creep. We consider a pipe with clamped ends ($\varepsilon_z = 0$). The ratio of the inner radius to the outer radius is $R_1/R_2 = 0.5$. The internal pressure is $P = 34$ MPa, and the external pressure is absent. The pipe is made of St. 45 steel, and the operating temperature is $T = 725$°C, i.e., the characteristics of the material are identical to those described above [see (13)–(15)]. We consider some statically possible stress fields, calculate the corresponding dependence $A_0 = A(t)$, estimate the rupture time t_i^*, and compare the results with the solution of the creep equations.

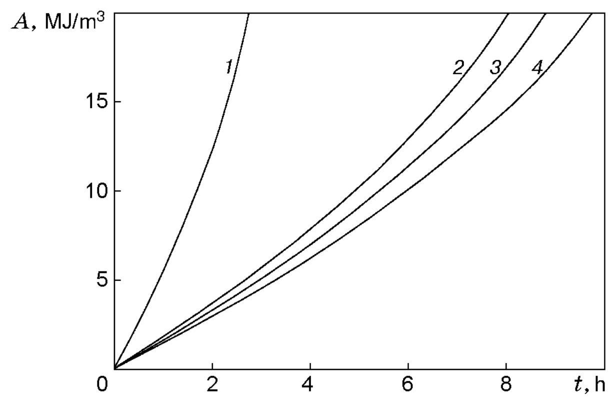

Fig. 5

1. We consider the stress field corresponding to the elastic solution as a statically possible stress field and assume that the stresses σ_r and σ_θ remain unchanged up to failure. Setting Poisson's ratio to be $\nu = 0.5$ and using the boundary conditions, we obtain the following relations [12]:

$$\sigma_r = \frac{PR_1^2}{R_2^2 - R_1^2} - \frac{PR_2^2R_1^2}{r^2(R_2^2 - R_1^2)}, \qquad \sigma_\theta = \frac{PR_1^2}{R_2^2 - R_1^2} + \frac{PR_2^2R_1^2}{r^2(R_2^2 - R_1^2)},$$
$$\sigma_z = \frac{1}{2}(\sigma_r + \sigma_\theta), \qquad \sigma_{\rm eq} = \frac{\sqrt{3}}{2}(\sigma_\theta - \sigma_r) = \sqrt{3}\,\frac{PR_1^2R_2^2}{r^2(R_2^2 - R_1^2)}. \tag{16}$$

Substituting (16) into the formula $W_0 = \dfrac{1}{V}\displaystyle\int_V B\sigma_{\rm eq}^n\,dV$, we find W_0 and, from (15), determine the relation $A_0 = A(t)$, which yields $t_1^* = 3.16$ h for $A_0 = A^* = 30$ MJ/m^3. Figure 5 shows the dependence $A_0 = A(t)$ corresponding to this statically possible stress–strain state (curve 1).

2. We use the limit stress field corresponding to the case of an ideal plastic body as a statically possible stress field. In this case, the stresses have the form [12]

$$\sigma_r = \frac{2}{\sqrt{3}}\sigma_0 \ln\frac{r}{R_2}, \qquad \sigma_\theta = \frac{2}{\sqrt{3}}\sigma_0\Big(\ln\frac{r}{R_2} + 1\Big), \qquad \sigma_z = \frac{1}{2}(\sigma_r + \sigma_\theta),$$

$$\sigma_{\rm eq} = \frac{\sqrt{3}}{2}(\sigma_\theta - \sigma_r) = \frac{2}{\sqrt{3}}\sigma_0, \qquad \sigma_0 = \sqrt{3}P(2\ln 2).$$

We find $W_0 = \dfrac{1}{V}\displaystyle\int_V B\sigma_{\rm eq}^n\,dV$. From (15), we determine the relation $A_0 = A(t)$ (curve 2 in Fig. 5), which yields $t_2^* = 9.2$ h for $A_0 = A^*$.

3. As the statically possible stress field σ_r, σ_θ, σ_z, we take the field corresponding to the "steady-state" solution and assume that these stresses remain unchanged from the beginning of creep to failure. After simple manipulations, we express $\sigma_{\rm eq}$ in the form [8]

$$\sigma_{\rm eq} = \frac{\sqrt{3}P}{(n-1)(4^{1/(n-1)} - 1)}\Big(\frac{r}{R_2}\Big)^{-2/(n-1)}.$$

As previously, we calculate W_0. From (15), we determine $A_0 = A(t)$ and $t_3^* = 10.1$ h (curve 3 in Fig. 5).

4. Considering the problem of unsteady creep of a pipe loaded by an internal pressure p, from the equations of strain compatibility and equilibrium, we obtain [13]

$$\frac{\partial\sigma_r}{\partial x} = \frac{\sigma_\varphi - \sigma_r}{x}, \qquad \frac{\partial\sigma_\varphi}{\partial x} = -\frac{\partial\sigma_r}{\partial x} - \frac{E}{1-\nu^2}\,\frac{\varepsilon_\varphi^c - \varepsilon_r^c}{x} + \frac{\nu E}{1-\nu^2}\,\frac{\partial\varepsilon_r^c}{\partial x} - \frac{E}{1+\nu}\,\frac{\partial\varepsilon_\varphi^c}{\partial x}.$$

This system supplemented by the conditions $\sigma_r(R_1) = -p$ and $\sigma_r(R_2) = 0$ is a boundary-value problem for determining the stresses $\sigma_r(r)$ and $\sigma_\varphi(r)$ for known creep strains ε_r^c and ε_φ^c and their derivatives at each point of the pipe at a certain moment t.

Combining the condition of proportionality between the stress deviators and creep strain rates and the equation of state in the energy form, we obtain a system of equations for determining the creep strains and specific dissipated work at each point of the pipe for the known stresses:

$$\frac{\partial A}{\partial t} = \frac{B\sigma_{\rm eq}^n}{(A^* - A)^m}, \qquad \frac{\partial \varepsilon_r^c}{\partial t} = \frac{\partial A}{\partial t}\,\frac{2\sigma_r - \sigma_\varphi - \sigma_z}{2\sigma_{\rm eq}^2}, \qquad \frac{\partial \varepsilon_\varphi^c}{\partial t} = \frac{\partial A}{\partial t}\,\frac{2\sigma_\varphi - \sigma_r - \sigma_z}{2\sigma_{\rm eq}^2},$$

$$t = 0, \qquad A = \varepsilon_\varphi^c = \varepsilon_r^c = 0.$$

The problem was solved by an iterative method. In each iteration, the stress distributions $\sigma_r(r)$ and $\sigma_\varphi(r)$ were determined by the shooting method for the boundary value of the radial stress using the strains calculated in the previous iteration. Then, the strain field was determined at the next time step. Curve 4 in Fig. 5 shows the dependence $A_0 = A(t)$ obtained by direct numerical calculations. For $A = A^*$, the rupture time is $t_4^* = 11$ h. The time of redistribution of the initial stress to the steady stress is 10 min, which is less than 2% of the duration of creep before failure.

Conclusions. The results of physical and numerical experiments can be summarized as follows:

1. The proposed approximate estimates of the intensity of processes of high-temperature creep and long-term strength of structural elements are in good agreement with experimental data.

2. The stress fields corresponding to the limit state of an ideal plastic body (or an ideal creep body) can be used as an approximate stress–strain state to estimate these processes.

3. In view of rapid distortion of twisted thin-walled tubular specimens in experiments, it is recommended to use thick-walled or even continuous specimens or specimens with a very small central axial channel to determine the shear characteristics of high-temperature creep. At high temperatures, experimental results obtained for thick- and thin-walled specimens are close.

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